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The mineral economy: a model for the shape of oil production curves

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Abstract

The production and the depletion of mineral resources, and especially oil and fossil fuels, has been an object of extensive predictive modeling. These predictions are often derived from Hubbert's model which is based on the fitting of the experimental data to a symmetric, bell-shaped curve. Although this model describes several historical cases, in particular, crude oil production in the lower 48 US states, not all theoretical models for the "mineral economy" are based on symmetric curves. Also, not much attention has been dedicated so far to the mechanisms which lead to such a behavior. In particular, scarce attention has been dedicated to the factors which may make the production curve asymmetric, e.g. a decline in production more abrupt than the growth. In the present paper, the author uses a stochastic model to examine factors affecting these phenomena. The results of the simulations indicate that the production curves of a non-renewable resource may be asymmetric in dependence on factors such as the search strategy or the presence of technological improvements. Considering worldwide oil production, the simulations indicate that the after-peak downward slope might turn out to be considerably more steep than the upward slope, something that could have unpleasant effects on the economy.

Keywords: Hubbert; Oil depletion; Mineral economy

1. Introduction

In the coming decades, mankind will be facing a problem which could turn out to be serious or even dramatic, that of the depletion the main source of primary energy in the world: fossil fuels and in particular oil. In order to plan ahead for the future, it is obviously important not only to know the amount of the ultimate recoverable resources (URR) of fossil fuels but also to use appropriate models to forecast the trends of extraction. The present paper addresses the problem by developing a general extraction model for the "mineral economy", i.e. for non-renewable resources.

The first model dealing with the concept of mineral economy was described by Hotelling in his classic 1931 paper (Hotelling, 1931). In more recent times, the depletion of planetary resources resulting from human use has been extensively modeled, for instance by the well-known Forrester/Meadows 1972 study "the limits of growth" (Meadows et al., 1972). Another approach was the one developed by Hubbert who, in the 1960s, modeled crude oil production in the United States (Hubbert, 1962) using an empirical approach based on a bell-shaped curve. The model was spectacularly successful in predicting the peak of oil extraction in the US lower 48 states and the subsequent decline in production. Symmetric, "Hubbert-like" production curves have been observed also for other cases, such as the production of anthracite carbon in Pennsylvania (Deffeyes, 2001).

Recently, several authors used Hubbert's model to predict the evolution of crude oil extraction at the planetary level (Deffeyes, 2001; Bentley, 2002; Campbell and Laherrere, 1998; Campbell, 2002). A feature of this kind of modeling is that depletion is gradual (indeed it may be argued that production will never actually go to zero Houthakker, 2002). The "production peak" is therefore the main event in the future history of oil extraction, a point which will mark the epochal change from cheap oil to expensive oil (Campbell and Laherrere, 1998). According to Hubbert, the production peak takes place at "midpoint", i.e. when half of the URR have been used up. Several estimates indicate that we are close to having extracted one trillion barrels of oil out of an original endowment of some 2 trillions "conventional" oil (Campbell, 2002). Hence, according to these

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estimates, the corresponding production peak could take place within the first decade of the 21st century (Deffeyes, 2001; Bentley, 2002; Campbell and Laherrere, 1998; Campbell, 2002) or not much later.

However, it is not obvious that Hubbert's model, successful for describing several cases of *local* resource depletion, can be automatically considered appropriate for describing the *global* case. Indeed, Hubbert's symmetrical curve is different from the typical curves of the Forrester/Meadows models (Meadows et al., 1972) which are almost always asymmetric with the decline much sharper than the growth. Further cases of asymmetric theoretical curves, characterized by an abrupt after-peak decline, are those presented by Wood and Long (2000). This possible shift of the peak away from midpoint has obviously important consequences in the future scenarios of fossil fuel depletion. If the downward slope will be much sharper than expected, the result could be an unpleasant (to say the least) shock for the economy.

In the present paper, I will discuss models for the extraction of mineral resources with the specific aim of determining what are the conditions that may lead to symmetric or asymmetric production curves. For this purpose I will develop a specific stochastic simulation in order to figure out how human policies may affect the shape of the curves with a special view to oil production and depletion. The model described here is inspired mainly by a model developed by Reynolds (1999) which is based on the concept of Robinson Crusoe surviving in a deserted island on hardtack tins left around by previous shipwrecks.

The calculations reported here show that there is no magic in the midpoint of the production curve. The bell-shaped curve may turn out to be strongly asymmetric depending on extraction strategies. By concentrating resources on extraction, the increased recovery of a non-renewable resource must be paid by a more rapid peaking and by a more abrupt fall afterwards. In practice, improving oil extraction technology may turn out to be counterproductive, just as improved fish catching techniques may have worsened the problems of some fisheries (Klyashtorin, 1998).

2. Models for the mineral economy

Complex systems can be often (although by no means always) described using simple mathematical models, i.e. models which use a small number of parameters. For instance, bell-shaped (e.g. Gaussian) and logistic curves describe a variety of different phenomena from traffic trends to atomic-scale phenomena. A model based on a manageable number of parameters can be said to be "robust", or to have "robust predictive characteristics". Such models may not be perfectly accurate, but will rarely be completely off the mark. Hubbert's empirical model for mineral extraction is certainly one of these robust models. In the form, for instance, of a Gaussian curve, the fitting to the experimental data depends only on two parameters.

A different approach to modeling may be based on the explicit description of interactions among the different elements of the model. Such models use a large number of parameters but may not be less robust than empirical ones. It is the case, as an example, of statistical thermodynamics which is based on small-scale models of interatomic interaction and which eventually emerged as parallel and equivalent to the purely empirical classical thermodynamics. Considering economics models, a model may assume specific laws which link "agents" operating in the model with the resources available. In the case of the mineral economy this is, essentially, the original approach of Hotelling (1931). Hotelling's model, however, fails to describe the historical trends of crude oil extraction, mainly because it neglects the dynamic interaction between production and economic growth.

There exist several models described in the literature which can give rise to "bell-shaped" extraction curves for a non-renewable resource. The basic approach here can be considered the classic "Lotka–Volterra" model, known since 1925 (Lotka, 1925; Volterra, 1926), also known as "Foxes and Rabbits" model. Here, the populations of foxes and rabbits are dynamically linked to each other and their interaction causes a cyclical behavior in the number of individuals of each. The model can also be applied to the case of the extraction of non-renewable resources assuming that the rabbits do not reproduce. In this case, it produces a single bell-shaped curve for the number of rabbit catches (i.e. production), not unlike the one predicted by Hubbert. This model is the historic basis of the system dynamics approach. It has the limitation that costs are not directly taken into account, something that may explain why it is not so popular with economists.

Recently, Reynolds (1999) proposed an approach for the mineral economy which also reproduces the "bell-shaped" Hubbert curve. Reynolds's model is described in terms of "Robinson Crusoe and the hardtack", a concept inspired, in turn, by the "Mayflower problem" described by Page (1979). The idea of Reynolds's model is that Robinson Crusoe's only source of food in his island are tins of hardtack left around by previous shipwrecks and buried in the sand. Assuming that no more shipwrecks take place after Crusoe's one, hardtack tins are a non-renewable resource. Depending on the total amount of hardtack and on his ability to find it, Crusoe may survive for only a short time or die of old age with still plenty of resources left. In Reynolds's model there is no extraction cost for the resource (or, at least, this cost is constant). The variability, i.e. the main factor in the cost of a tin, is the time that Crusoe spends searching for it. In some conditions this cost may turn out to be too high for him to afford. Hence he may die of starvation. Reynolds' calculations generate bell-shaped production curves in some specific assumptions, i.e. depending on Crusoe's capability to learn more efficient search strategies. However, the model neglects, just as Hotelling's model does, the dynamic interaction of resource extraction with the economy that is benefiting from this extraction.

The model developed here is a combination of the two models just described. It is inspired mainly by Reynolds's one (Robinson Crusoe searching for buried hardtack) but, as in the Lotka–Volterra model, it assumes population growth in the island (let us say that Crusoe's companion, Friday, was a girl). In the present model, population growth may be simulated by the standard assumption that the variation of population is directly proportional to population (Barnett, 1962), an assumption that gives rise to an exponential population growth as long as there exist resources of food available. This assumption simulates economic and population growth caused by the resource being extracted. It is close to the real-world situation since both the economy and population have been growing exponentially during the past century or so. This growth may be related to the abundance of oil and natural gas, which are powering the economy and are used to manufacture fertilizers and fuel to transport food worldwide.

The other feature of the model is the definition of the cost of the resource (hardtack). Following Reynolds, it is assumed here that hardtack tins are buried under a shallow layer of sand. We assume also that islanders perform cycles of search. For each cycle, the probability for an islander of finding a buried tin of hardtack can be assumed to be

$$p = k(t)(N - N(t))/N, \tag{1}$$

here N is the initial endowment of hardtack in the island, i.e. the hardtack URR. $N(t)$ is the cumulative number of tins found by the islanders and $k(t)$ is an "ability factor" which describes the average capability of the islanders of finding hardtack. In the simplest version of the model, $k(t)$ may be taken as a constant and as unity for convenience. However, it may vary with time when, for instance, islanders learn better search strategies. In the formula, "p" is proportional to the amount of hardtack found in a single search. In each search cycle, the number of searches divided by the number of hardtack tins found is the cost of a unit of resource (a tin). It follows that the cost of the resource is inversely proportional to the probability to find it.

The model as described might be considered over-simplified since it assumes that all tins are of the same size and that the only cost is related to locating them and not to extracting them. This is in contrast to the known situation of oil fields, where the cost of extraction varies widely in relation to the field size. Furthermore, it might be argued that most oil fields have been already located and that at present the main cost of crude oil is related to extraction.

These objections carry a weight but one should also take into account that models are always approximations and that one of the crucial features for a model to be useful is that the number of parameters involved should be manageable. It would not be difficult to incorporate in the model we are considering a function which describes different extraction costs assumed to vary depending on the size of a find (variable number of tins found together) and on the depth of burial. Doing this, however, we would have to introduce several arbitrary assumptions and parameters. The model might gain realism but it would lose generality and robustness. As we said, the power of Hubbert's model lies in the fact that it considers only two parameters and that these parameters are not related to a specific distribution of the sizes of oil fields.

A further consideration on this point is that the mathematical formulation of the model we are developing is abstract and there is considerable latitude in the way we may see it as approximating some real-world situation. The essential point of the model is to establish a "cost" for obtaining a unit of resource, and that this cost increases as the resource becomes scarce. Reynolds's describes this cost in his model as related to the a search time needed to find it. However, the same concept may very well be seen as related to the time needed to *extract* a progressively more deeply buried resource. The mathematical function that was defined proportional to cost, $(N - N(t))^{-1}$ may be considered as a perfectly reasonable description of a "cost of extraction" that increases asymptotically to infinity as islanders are forced to dig out more deeply buried tins of hardtack. For the same reasons, the $k(t)$ factor may be seen as the result of better *searching* technology, but just as well of better *extraction* technology. To maintain things simple (and robust) as much as possible, in the following the model will be described and interpreted as in the original Reynolds' approach, e.g. with the cost defined as due to searching only. We should keep in mind, however, the alternative interpretation of cost due to extraction and such interpretation will be mentioned when it appears appropriate.

The model was implemented using a stochastic (or "Monte Carlo") calculation. In the simulation we assume the presence of a starting number of islanders, or "agents" (at least one) and of an initial endowment of hardtack tins, or "resource units". The simulation runs in cycles which, given the characteristics of the simulation, should be intended as representing at least a few years each. However, the actual time length of a cycle is simply a scale factor that does not affect the results of the simulation.

For each cycle of the simulation the islanders perform a number of search attempts. As described before, each search has a probability of success which is assumed to be directly proportional to the amount of hardtack left on the island. Within each cycle, islanders have a chance of finding several tins or, equivalently, tins of different size.

If the islander finds hardtack, he (let us assume that males only are active in the search) stores it as his own property. Every cycle, each islander has a chance to reproduce, generating another islander who inherits half of the father's hardtack. In every cycle, the islanders will also use some of their hardtack as food. An islander is removed from the simulation (dies of starvation) if the amount of hardtack he owns goes down to zero. As the simulation progresses and the amount of hardtack dwindles, all the islanders must eventually die off.

These are the basic features of the model, which can be subsequently modified to take into account more details and variations. The main variants considered here are described as follows:

1. *Fixed time search:* The islanders perform a fixed number of searches in each cycle, accumulating all the hardtack they find (this is the ''basic'' model described before).
2. *Optimal amount search:* In each cycle the islanders continue to search until they collect an amount of hardtack that they judge optimal for their needs. (the maximum number of searches is anyway finite).
3. *Technology factors:* The probability of success in the search improves with the success of previous searches. In other words, islanders learn better search strategies.
4. *Time lag search:* In this case, the located hardtack tins become part of the islander's hoard only after a certain number of cycles. In other words, there is a delay between identification and production of the resources.

The first two models differ in the strategy used by islanders but their aim may be assumed to be the same, i.e. to maintain a constant flux of resources per each islander. In the ''fixed cost'' model, islanders keep expending the same amount of effort (that can be seen as either time or money) in their search regardless of the results. As the simulation runs on, the amount of hardtack they collect gradually dwindles and eventually goes to zero. This way of searching may appear rather dumb, but it actually describes a perfectly reasonable strategy to keep constant the flux of the resource in the case in which there is a chance to switch to another source: let us say hardtack coming from another island. This feature may be explicitly simulated by providing islanders with a constant flux of ''extra-island'' hardtack. Therefore, this model can be seen as describing the case of resource depletion in a specific area (as for instance the case of the US production dwindling and the supply source moving to the Middle East). Switching to other producers, prices remain approximately constant and from the viewpoint of users there is no depletion at all.

In the second case (optimal amount search), the islanders are still trying to maintain constant the flux of resources but in this case they increase their efforts as the supply declines and, as a consequence, hardtack tins become more expensive. This model is closer to the ''global'' case, when there is no other source than the one being exploited. Since the search time available to islanders is in any case limited, as the simulation progresses hardtack tins become more expensive and eventually the islanders will be no more able to afford them.

The ''technology'' (model 3) parameter is something that affects the search probability taking into account the capability of islanders to learn more efficient search strategies. In Reynold's model (Reynolds, 1999), two technological parameters are defined. One is the short-term ''Q'' and one the long-term ''L'' in the form of a multiplier of the probability $\exp(-Q/L)$. Q reduces the probability of finding a tin in each successive search the same cycle. It represents the expenditure of ''information capital'' as each islander exhausts areas of search where the available data tell him that finding hardtack is likely. Q restarts at zero at each cycle and increases proportionally to the amount of hardtack found in that cycle. ''L'' is instead a long-term parameter which increases with each cycle. It simulates the increasing technological capability of islanders to figure out where shipwrecks may have taken place, to locate debris that indicate a possible shipwreck, etc. In Reynolds' simulation the L parameter is defined as a function of time as a Hubbert-style bell-shaped curve.

I have tested both concepts (short and long-term effects on probability) in the present simulation. The first one, ''short term'', was simulated assuming that tins are ''clustered'' near each other; finding a tin increases the probability of finding others immediately afterwards. For the second one, long term, I have not used Reynolds's function as it seems too much of an ''ad hoc'' assumption. It seemed simpler and more realistic to assume that the ''technology factor'' is a simple linear function of the amount of previously found hardtack. The ''technology'' parameter in the present calculations is therefore simply a multiplying factor of the probability that starts as equal to one and increases proportionally to the total amount of found hardtack. Two cases were simulated, one is where each islander has a specific ability that depends on his personal past success, the other in which the islanders share their knowledge with each other and the average ability progresses with the average amount of hardtack found.

The technology factor can be implemented together with either the ‘‘fixed time search’’ model or the ‘‘optimal amount search’’ one. It should be remarked that the assumption that better technology increases the amount of discovered oil may not be really consistent with the real-world situation. However, the effect of this parameter was explored anyway since an opinion that appears in the literature is that better exploring and/or drilling technology may be a solution of the problem of oil depletion (see e.g. Lynch, 2002).

The ‘‘time lag’’ model (model 4) may be visualized assuming that islanders use some kind of metal detector to locate tins. As they find one tin, they plant a flag on the spot to come back later to dig it out. This is simulated assuming that found tins become available as part of the islander's hoard only after a fixed number of cycles. The model can be coupled with other assumption such as technology improvements and/or optimal amount searches. Obviously, here, the decision for the islander to expend more effort on searches will depend on the actual amount *produced* in that cycle, not on the amount *located*.

Summarizing, here are the basic parameters common to all calculations:

1. starting number of islanders
2. starting number of hardtack tins
3. probability of finding a hardtack tin
4. number of searches per cycle
5. threshold of accumulated hardtack needed for reproduction
6. probability for each islander to reproduce in each cycle

Other parameters specific for the variants and are:

7. optimal amount of hardtack to be found per cycle (optimal amount search)
8. multiplying factor for ‘‘technological ability’’ (individual/collective, long/short term)
9. number of delay cycles (‘‘time lag’’ model)
10. number of extra searches to be performed in each cycle for low hardtack production (‘‘time lag’’ model)

3. Results and discussion

The model just described is simple and can be implemented using any programming language. Starting with one islander and 32,000 tins of hardtack gives rise to a growth of a few hundreds of islanders in 50–100 cycles, which seems to be statistically significant since the overall shape of the curves does not change for larger numbers. For the same parameters, the simulations were found to be reproducible within some limits (an example of reproducibility is shown in Fig. 1). The main effect of the random factors in the stochastic modeling was found to operate in the initial stages of the simulation. However, the overall shape of the curves was scarcely affected. This dependence on the initial conditions seems to be typical of stochastic simulations, see for instance (Acemoglu and Zilibotti, 1997).

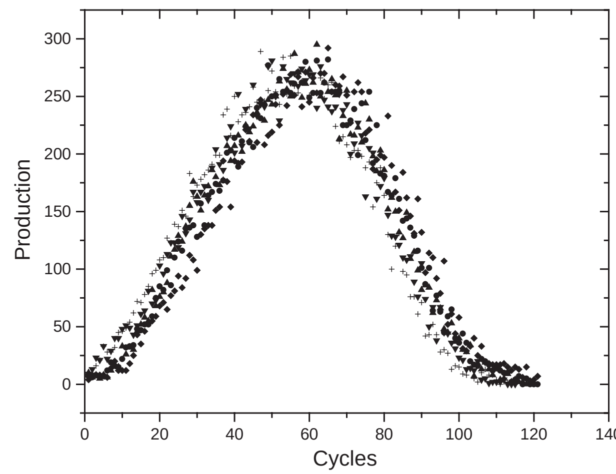

Fig. 1. Comparison of five simulation runs for the ‘‘fixed time search’’ model starting with the same parameters. This figure shows the dispersion in the results which is mainly due to fluctuation in the initial cycles.

The results of the simulations are, in general, in agreement with what one would intuitively expect. In all calculations we have an initial nearly exponential growth of hardtack (oil) production and of the island population. Both are followed by a subsequent decline and the final death of all the islanders. Only in the case of the assumption of no reproduction (‘‘Crusoe alone’’, i.e. population of constant size) the simulation reverts to Hotelling's original model, with an exponential decay of hardtack production and an abrupt collapse of the population after reaching zero production.

The effects of the ‘‘basic’’ parameters common to all models (1–6, see previous section) turned out to be mainly factors of scale, not significantly affecting the shape of the calculated curves. Therefore, their effect will not be further described in the following. In all cases, except when the probability of finding a tin was fixed as unity, the results of the simulations showed that only part of the URR was recovered, i.e. the islanders died out with some (and in some cases substantial amounts of) hardtack still left buried in the islands. The actual recovery fractions vary in dependence of the parameters used and will not be detailed here.

Models, to have any relevance, must be able to reproduce, at least qualitatively, the experimental data. In this case, the crucial test of the model is its capability to reproduce the approximately symmetric production curves known for historical cases of *local* mineral extraction. The model produced nearly symmetric production curves in the hypothesis of "constant cost" of the resource ("fixed time search"). This condition approximates that of a market where customers switch to remote producers when local production dwindles because of depletion. If the model was modified taking into account "extra-island" hardtack, the shape of the production curve did not substantially change although, of course, the population kept increasing exponentially. Typical results for this model are shown in Fig. 2. The calculated production curves are never exactly symmetric but always slightly skewed forward. Nevertheless, the asymmetry is small, as quantified in Table 1. The fitting of the simulated production curve with a Gaussian function is good, as shown in Fig. 3, just as

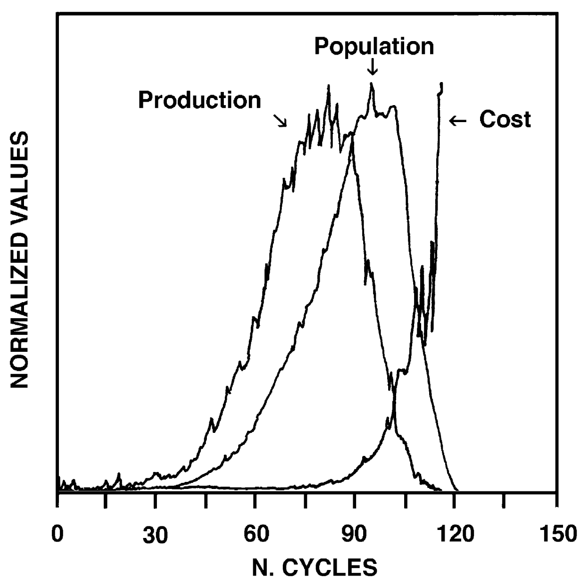

Fig. 2. Typical production, population, and cost curves for the "fixed time search" model described in the text. In this case the production curve is nearly symmetrical, as in Hubbert's model.

Table 1
Skewness of the curves obtained in the simulation analyzed using the standard definition of the term (see for instance Press et al., 1988), i.e. as the third momentum of the distribution

Model	Skewness
Fixed time search	0.85 ± 0.03
Optimal amount search	1.16 ± 0.03
Fixed time search and ability factor increasing from 1 to 3.5	1.50 ± 0.02
Fixed time search and ability factor increasing from 1 to 30	1.76 ± 0.03

The values reported in this table have been obtained for a series of simulation runs where all the parameters were kept fixed for the various models except for those specifically described as variable. This set of values can be considered representative of a larger set of calculations for different parameters; however, owing to the stochastic nature of the calculations the numeric values of the results, i.e. the skewness, are variable.

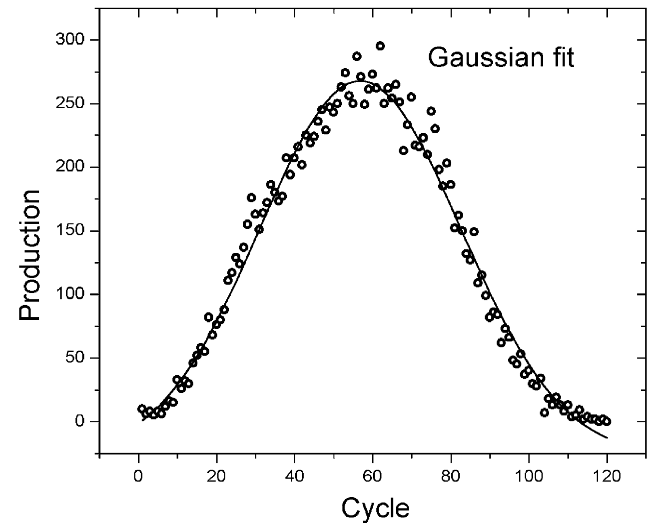

Fig. 3. Fitting of the results of a "fixed time search" (see text) simulation with a Gaussian function.

experimental production curves can be well fitted with a Gaussian curve (see, e.g. Deffeyes, 2001). The model does therefore qualitatively reproduce the historical trends, for instance the bell-shaped production curve of oil production in the US lower 48 states (Fig. 6), for assumptions which are consistent with the real market situation.

After this credibility test, we may move onwards and see how different hypothesis modify the trends. In the "global" case, consumers of the resource have no possibility of switching to another source (no close-by island in the simulation, no close-by earthlike planet in our case). In this case, in order to maintain a supply of resources sufficient for their needs, the islanders in the simulation must step up their search effort, i.e. they must be willing to pay more per resource unit. The model that takes into account these hypotheses is the one defined "optimal amount search". This simulation leads to a more asymmetric curve than the case described before, with the decline sharper than the growth (see Table 1 for quantification). A graphical comparison of the two models is shown in Fig. 4. In this case the fraction of recovered resource increases, but this increased recovery must be paid with a more rapid decline after the peak.

The parameters that simulate technological improvements can be introduced in both search strategies described above. "Short term" and "long term" parameters (as described in the previous section) were found to give similar effects. The calculation showed also that there are no significant differences in the final results for the two cases in which the "individual" or the "overall" ability increases were modeled. In all cases, if the improvement in the searching ability leads to a larger amount of hardtack, the fall occurs earlier and more rapidly. Fig. 5 shows an example for the simple case of

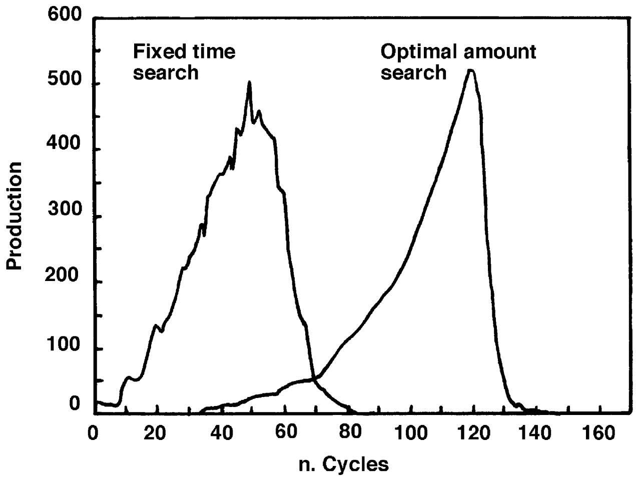

Fig. 4. Comparison of the "fixed time search" or "optimal amount search" strategies. The leftmost curve (fixed time) is the nearly symmetrical "Hubbert-like" curve calculated as described in the text, assuming that islanders search at random the island for a fixed time and accumulate all the hardtack they find. The other curve (optimal amount) assumes that they stop searching after they have accumulated an optimal number of hardtack tins.

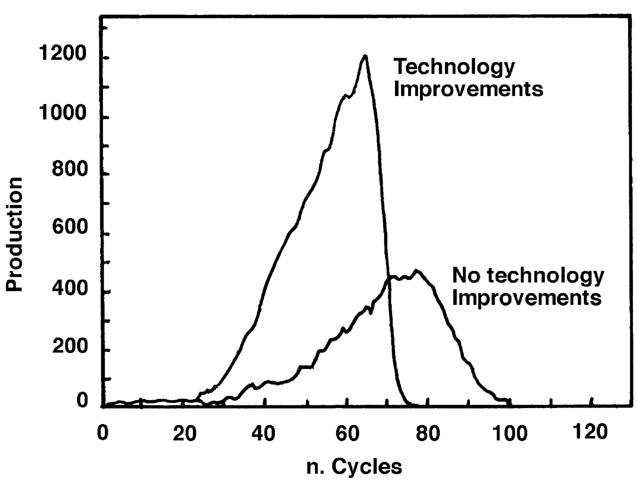

Fig. 5. Comparison of the "fixed time" search strategy with and without the parameter that assumes a linear progressive increase in the capability of the islanders to find hardtack, i.e. technological improvements.

an average linear increase of the long-term ability parameter (see also Table 1). In general, all parameters which improve the search efficiency of the islanders, e.g. the number of searches performed in each cycle, lead to more asymmetric peaks which decline more rapidly. The limit of this approach is to assume "perfect" technology, i.e. one that reduces to zero the search cost all over the simulation. In this case, the curves become sawtooth shaped with an abrupt fall in both production and population.

The final set of parameters considered are those which were described in the previous section as "time lag" search, i.e. the assumption that the located hardtack

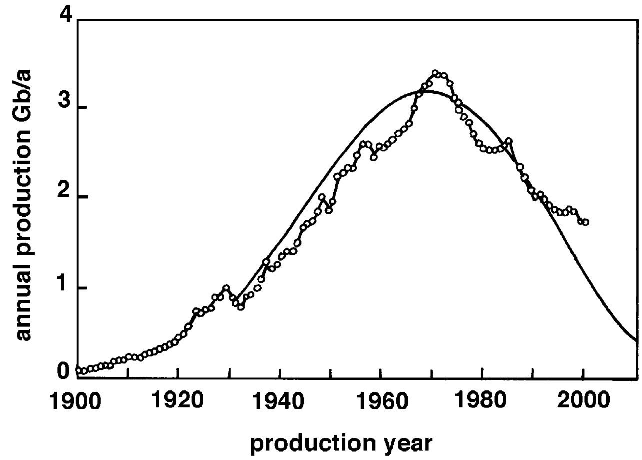

Fig. 6. Data about oil production in the lower 48 US states. Reproduced with permission from Barnett (1962).

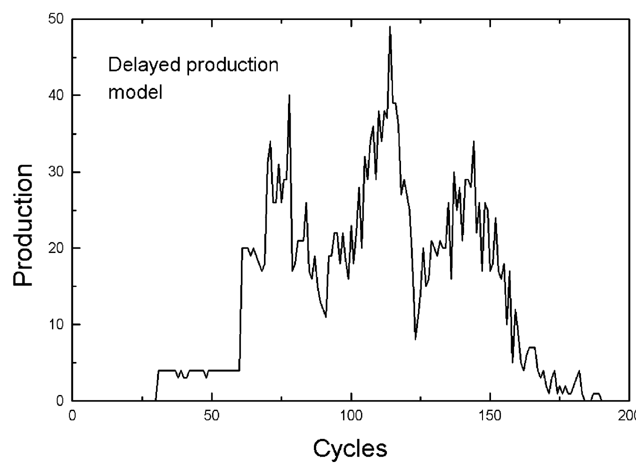

Fig. 7. An example of calculation for the "delayed production model" coupled with the assumption of extra searches performed for a low threshold of production. In this case, the resources are assumed to be actually produced after 30 cycles from discovery and the result are three cycles of production.

becomes available only after a certain number of cycles. This assumption leads to a considerable increase in the complexity of the model. In practice, the results obtained can be summarized as follows: (1) If the time delay assumption is coupled with the simple "fixed time search" strategy, there are no significant differences in the shape of the curves in comparison to the simpler models. The only difference appears to be a higher sensitivity to the initial factors so that random fluctuations may lead the production curve never to pick up speed and the population collapsing to zero in the very early stages of the simulation. (2) if the time lag is coupled with the "optimal search strategy" it is possible to observe oscillations in the production curve (Fig. 7), an effect that appears to reproduce the oscillations of some real cases such as the oil production in Russia

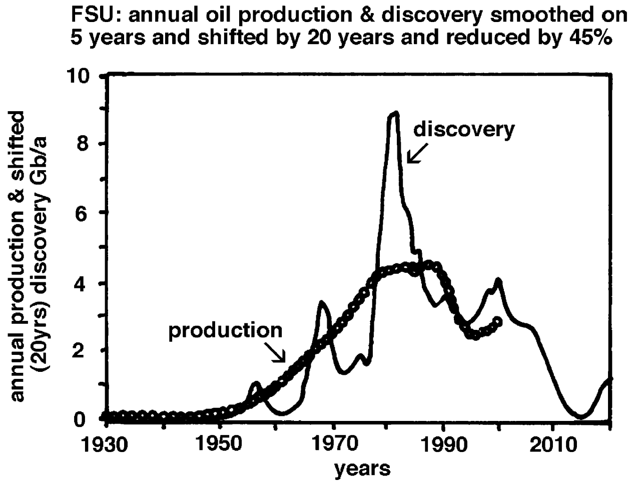

Fig. 8. Data for oil production in the former Soviet Union, Reproduced with permission from Barnett (1962).

(Laherrère, 2002) (Fig. 8). The number of parameters to be used in this kind of simulations is, however, too large to define specific trends.

Finally, regarding costs, the present simulations confirmed the results of Reynolds (1999). For the simplest models (fixed time search and optimal amount search), costs increased throughout the simulation, with the rise highly noticeable after the production peak. If the "technology" parameter was introduced, costs could actually go down before the peak before rising sharply afterwards.

The main results obtained follow robust trends which can be summarized as follows:

1. A "bell-shaped" curve of production of a mineral resource is always obtained except for very special assumptions. However, the curve is not necessarily symmetric.
2. There is no magic in the "midpoint" of the production of a mineral resource. Depending on the search and/or extraction strategies, the production curve may be asymmetric with the peak shifted forward in time and with a decline much more rapid than the growth.
3. The results reported by Reynolds (1999) are confirmed: prices and costs can falsely signal decreasing scarcity. Only near the maximum of production, prices will start a rapid rise as an effect of scarcity. If "technology" is taken into account, prices may actually fall before the production peak is reached.
4. In no case, except for very special assumptions, the simulations produced total recovery of the mineral resource. In other words, as pointed out by Houthakker (2002), there may be no such thing as "exhaustion" of a mineral resource. Rather, at some point it will become too expensive to be worth extracting. This may be, of course, a problem if it is a crucial primary resource such as crude oil.
5. Increased efforts or improved technologies increase the amount of resource recovered, but this increased recovery must be paid for with a more abrupt fall after the peak.

These results indicate that Hubbert's model is a good approximation but that in some cases it may not be enough for an accurate prediction of crude oil production. The model would need to be modified taking into account a new parameter: the "skewness" of the bell-shaped curve. Of course, adding a further parameter increases the complexity of the theory and therefore makes quantitative forecasting more difficult. Nevertheless, the conclusions derived from such a model may be qualitatively relevant. Literature estimates based on Hubbert's model indicate that the worldwide peaking of crude oil production could take place in less than a decade from now (Deffeyes, 2001; Bentley, 2002; Campbell and Laherrere, 1998; Campbell, 2002), even though other estimates forecast a later peaking (Wood and Long, 2000). The results of the present work do not change the URR estimates on which these predictions are based. However, they indicate that in some conditions peaking may be delayed and the growth of production may be sustained beyond the "midpoint" of resource consumption. This delay in peaking, however, may be paid by a more rapid decline after the peak, something that might be a dramatic shock for the economy.

4. Conclusion

No matter how sophisticated simulations can become, the future remains obscure and computers may not be better at predicting it than more ancient methods such as oracles or prophetic dreams. It cannot be proved that the result reported here, valid for a "toy world" of islanders seeking for buried hardtack, can be applied to the much more complex real world. However, even though these models may not be quantitative in their predictions, they can be nevertheless an aid to devising policies. For the present set of calculations, looking from "God's viewpoint" (or the programmer's) it is easy to see that the islanders are making a bad mistake in concentrating all their efforts in searching harder and harder for hardtack. Developing more efficient "hardtack finding technology" gives them an illusion of abundance and low prices, but it makes them vulnerable to the shock of the abrupt fall that takes place after the production peak is reached. They ought, rather, start making fishing tackle. But we can almost hear the tribal elders muttering: "hardtack resources are still abundant'', ''fish resources are too distributed'', ''how are you going to solve the fish storage problem?'' and the like. For the real inhabitants of the planet earth, concentrating all efforts in better methods for extracting fossil fuels, rather than developing alternative sources, could be the same kind of mistake.

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References

Acemoglu, D., Zilibotti, F., 1997. Was Prometheus unbound by chance? Risk, diversification, and growth. Journal of Political Economy 105, 709–751.

Barnett, V.D., 1962. The Monte Carlo solution of a competing species problem. Biometrics 18, 76.

Bentley, R.W., 2002. Global oil & gas depletion: an overview. Energy Policy 30, 189–205.

Campbell, C., 2003. The Essence of Oil & Gas Depletion. Multiscience Publishing, Brentwood.

Campbell, C.J., Laherrère, J.H., 1998. The end of cheap oil. Scientific American, March, 60–65. See also the internet site www.hubbertpeak.com.

Deffeyes, K.S., 2001. Hubbert's Peak, The Impending World Oil Shortage. Princeton University Press, Princeton.

Hotelling, H.J., 1931. The economics of exhaustible resources. Journal of Political Economy 39, 137–175.

Houthakker, H.S., 2002. Are minerals exhaustible? The Quarterly Review of Economics and Finance 42, 417.

Hubbert, M.K., 1962. National Research Council publication 1000-D, Washington, DC, p. 54.

Klyashtorin, L.B., 1998. Long-term climate change and main commercial fish production in the Atlantic and Pacific. Fisheries Research 37, 115.

Laherrère, J.H., 2002. Is FSU oil growth sustainable. Petroleum Review April, 29–35.

Lotka, A.J., 1925. Elements of Physical Biology. Williams & Wilkins Co., Baltimore.

Lynch, M.C., 2002. Forecasting oil supply: theory and practice. The Quarterly Review of Economics and Finance 42, 373–389.

Meadows, D.H., Meadows, D.L., Renders, J., Behrens III, W.W, 1972. The Limits of Growth. Universe Books, Geneva.

Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., Numerical Recipes in C: The Art of Scientific Computing, 1988. Cambridge University Press, Cambridge.

Page, T., 1979. Conservation and Economic Efficiency. The Johns Hopkins University Press, Baltimore.

Reynolds, D.B., 1999. The mineral economy: how prices and costs can falsely signal decreasing scarcity. Ecological Economics 31, 155.

Volterra, V., 1926. Variazioni e fluttuazioni del numero di individui in specie animali conviventi. Memorie della Regia Accademia Nazionale dei Lincei 2, 31–113.

Wood, J., Long, G., 2000. Long term world oil supply: a resource base/production path analysis. http://www.eia.doe.gov/pub/oil_gas/petroleum/presentations/2000/long_term_supply/index.htm.